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Debt and Warrants: Agency Problems and Mechanism Design*

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This paper shows that a debt contract with warrants for the lender and cash/equity settlement options for the entrepreneur–borrower is the optimal contract in a setting with moral hazard and unverifiable states of nature. This contract makes it possible to contract optimally ex ante on *unverifiable* states; debt obligations are adjusted to state realizations so that debt is made safer. Moral hazard is reduced as a result. *Journal of Economic Literature* Classification Numbers: 026, 520. © 1992 Academic Press, Inc.

I. INTRODUCTION

This paper provides a new rationale for the role of warrants in incentive contracting. I examine a principal–agent model of financial contracting in which a risk-neutral entrepreneur (agent) undertakes a project and makes an unobservable effort choice while employing the funds of a risk-neutral lender (principal). The entrepreneur's liability is limited to the project's return; the distribution of this return depends on the entrepreneur's effort

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as well as on the realization of the state of nature.[1] The state realization is observed both by the entrepreneur and by the lender at some interim date, before the effort choice has to be made. States of nature cannot be contract upon ex ante, because state verification by third parties is impossible.[2] The problem I examine is then one of moral hazard with unverifiable states. My main result is that the optimal financial arrangement is a debt contract with warrants for the lender and cash/equity settlement options for the borrower (entrepreneur).

The optimality of this contract derives from the presence of state-contingent debt-overhang problems. The worse the state realization, the smaller the entrepreneur's return to effort for any given face value of the debt. Consequently, with straight debt, the entrepreneur underinvests in effort more in bad states than in good states; thus, straight debt leads to state-dependent inefficiencies. State-contingent debt obligations can remedy such inefficiencies by calling for greater debt payments in better states, thereby at least partially attenuating the underinvestment problem. The impediment to implementing such an arrangement is that states of nature cannot be contracted upon ex ante. In such an incomplete information environment, it turns out that attaching warrants to debt for the lender and cash/equity settlement options for the borrower is the ex ante optimal scheme. This allocates higher debt payments to better states; this is achieved by permitting the lender, upon observing the realization of the good state, to exercise the warrants prior to the effort choice. The entrepreneur is then forced to either dilute his equity holdings or make the cash settlement. He chooses the latter and by so doing effectively makes the high debt payments that are required in good states. The optimal resource allocation is attained as a result.

The above solution reduces moral hazard by imposing losses on the lender in bad states that are offset by the high debt payments that are made in good states. This is feasible because contracting occurs before the state realization, and hence the lender's participation constraint needs to hold in an ex ante sense. In contrast, if lending takes place after the state realization, then the lender's participation constraint must be satisfied in *that* state, implying that spot lending adds additional constraints to the entrepreneur's optimization problem. Hence, even when the funds are not needed until the state is realized, the optimal arrangement calls for contracting prior to the state realization.

[1] This is done to ensure that the optimal contract takes a form other than debt. Innes (1990) shows that, in a setting like the one here, the optimal contract is debt if the project return distribution depends only on effort.

[2] States of nature cannot be described ex ante in a sufficiently clear and unambiguous way so that an outsider (e.g., courts) can judge ex post which state has occurred. For a review of the difference between observability and verifiability and its implications for contract design, see Hart and Holmstrom (1987).

The most closely related papers are Green (1984), Brennan and Kraus (1987), and Constantinides and Grundy (1989). Green shows that warrants can ameliorate the asset-substitution moral hazard problem of risky debt (also see Barnea *et al.*, 1985). However, in the setting of that paper, equity can achieve the first best as well. Both Brennan and Kraus and Constantinides and Grundy highlight the role of warrants in mitigating the costs of ex ante informational asymmetries. In this paper, I abstract from ex ante informational asymmetries to focus on the incentive effects of financing.

The structure of the paper is as follows. Section II describes the model and discusses the key ingredients of the contracting problem: moral hazard and the univerifiability of states of nature. Section III derives the optimal state-contingent contract, thus reducing the original problem to an implementation problem. This implementation issue is dealt with in Section IV, where the main results on the contractual form are presented. Section V shows that the results hold with an arbitrary number of states. Section VI concludes.

II. STATEMENT OF THE PROBLEM

A. *Agents*

There are two (risk-neutral) agents who enter into a contract: an entrepreneur who must raise funds to finance a project and a lender. For simplicity, we take the entrepreneur's initial wealth to be zero. The credit market is perfectly competitive. To simplify, we assume a market interest rate of zero.

B. *The Project*

The project requires: (i) a cash outlay of k at date $t = 0$ and (ii) an input of entrepreneurial effort, π, at date $t = 1$. The return, r, is realized at $t = 2$, the terminal date. The return distribution depends upon the entrepreneur's effort and the realization of the state of nature, Θ^j. The latter occurs with probability q_j and is observed by the entrepreneur and the lender as well as by market participants before the effort choice is made.[3] The time line below shows the sequence of events:

Contract is negotiated	The state of nature, Θ^j, is revealed	Entrepreneur's investment decision (input of effort, π)	Realization of returns, r
$t = 0$		$t = 1$	$t = 2$

[3] The date the cash outlay has to be made is a matter of indifference. What is relevant is that the structure of the problem is known before the state realization.

We make the following assumption. If the state is Θ^j, then

$$r \in \{r_s^j, r_f^j\}, \text{ with} \qquad r_s^j > r_f^j \geq 0, \qquad \forall j, \tag{1}$$

where r_s^j (r_f^j) denotes the return at $t = 2$ when the project succeeds (fails) in state Θ^j. We initially assume $j = 1, 2$, and then discuss the generalization to an arbitrary number of states.

By the choice of effort, the entrepreneur chooses the probability that, for any given state of nature, the outcome is a "success;" i.e.,

$$\text{prob}(r_s^j) = \pi^j, \tag{2}$$

where π^j denotes the entrepreneur's input of effort, π, in state Θ^j. The entrepreneur's (nonpecuniary) cost of effort, $C(\pi)$, is defined over $\pi \in [0, 1]$, with

$$C'(\pi) > 0, \quad C''(\pi) > 0, \quad \lim_{\pi \to 0} C'(\pi) = 0, \quad \lim_{\pi \to 1} C'(\pi) = \infty. \tag{3}$$

As we are interested in characterizing the solution to the entrepreneur's optimization problem, we assume that a solution exists. This is tantamount to assuming that the outcome "success" is sufficiently high that the project has positive net present value and therefore is worth financing.

We denote the good state Θ^2 and the bad state Θ^1, and assume

$$r_s^2 > r_s^1 \tag{4a}$$

and

$$r_f^2 \geq r_f^1. \tag{4b}$$

The problem is interesting when safe straight debt cannot be issued. We thus assume

$$r_f^1 < k. \tag{4c}$$

C. *Feasible Contracts*

The set of contracts which can be enforced is restricted because:

(i) The entrepreneur's input of effort can not be specified in a contract: it is either unobservable or unverifiable.

(ii) The state of nature at date $0 < t < 1$ is *observed* by both the entrepreneur at the market participants. However, as emphasized in the Introduction, it is not *verifiable* by outsiders, e.g., courts.

As a result of these two assumptions, contracts can be conditioned only upon the terminal payoff of the project.

D. *The First Best*

As a benchmark case, we derive the solution which would be attained if the entrepreneur had sufficient wealth to self-finance the project.

The entrepreneur solves the following problem at $t = 1$ in state Θ^j.

$$\max_{\pi^j} \{(1 - \pi^j)r_f^j + \pi^j r_s^j - C(\pi^j)\},$$

and thus sets his input of effort to the value, π_{FB}^j, which solves

$$C'(\pi_{FB}^j) = r_s^j - r_f^j. \tag{5a}$$

Since the effort cost satisfies (3), a unique, interior solution for effort exists.

The expected value of the project at $t = 0$, $E_0(V)_{FB}$, is

$$E_0(V)_{FB} = \sum_j q_j E(V|\Theta^j)_{FB}, \tag{5b}$$

where

$$E(V|\Theta^j)_{FB} = r_f^j + \pi_{FB}^j(r_s^j - r_f^j) - C(\pi_{FB}^j). \tag{5c}$$

The entrepreneur's expected payoff at $t = 0$, $E_0(W_E)_{FB}$, is then

$$E_0(W_E)_{FB} = E_0(V)_{FB} - k. \tag{5d}$$

III. STATE-CONTINGENT DEBT

This section derives the form of the optimal contract given the assumption that states of nature can be contracted upon ex ante. It provides an intermediate step for the main result on the contractual form that is derived in the next section. The main result of this section is that an optimal contract is a debt contract that allocates higher debt payments to better states.

Assume that states of nature can be specified in a contract. Then the entrepreneur's problem is to choose the face value of the debt, P^j, contingent on state Θ^j, and the effort, π^{ij}, to be taken at $t = 1$ under debt $(P^j) \equiv \min(r, p^j)$ in state Θ^j, $j = 1,2$, so that his expected payoff is maximized.

The entrepreneur's optimization problem can be stated as

$$\max_{P^j,\pi^{ij}} \left\{\sum_j q_j E(V|\Theta^j) - k\right\} \tag{6a}$$

s.t.

$$\sum_j q_j E(W_{\mathrm{L}} \mid P^j, \Theta^j) = k \tag{6b}$$

$$\pi^{ij}\text{: arg}\max_{\pi^{ij}} \{E(W_{\mathrm{E}}|P^j, \Theta^j)\}, \tag{6c}$$

where $E(V|\Theta^j)$ is the expected value of the project conditional on Θ^j,

$$E(V|\Theta^j) = (1 - \pi^{ij})r_{\mathrm{f}}^j + \pi^{ij}r_{\mathrm{s}}^j - C(\pi^{ij}). \tag{7}$$

$E(W_L|P^j, \Theta^j)$ and $E(W_E|P^j, \Theta^j)$ are the conditional expected payoffs to the lender and entrepreneur, respectively, under debt (P^j) in state Θ^j,

$$E(W_L|P^j, \Theta^j) = (1 - \pi^{ij})\min(r_{\mathrm{f}}^j, P^j) + \pi^{ij}\min(r_{\mathrm{s}}^j, P^j), \tag{8}$$

$$E(W_E|P^j, \Theta^j) = (1 - \pi^{ij})\max(r_{\mathrm{f}}^j - P^j, 0) + \pi^{ij}\max(r_{\mathrm{s}}^j - P^j, 0) - C(\pi^{ij}) \tag{9}$$

The above formulation states that the entrepreneur's problem is to choose P^j and π^{ij} so as to maximize the net present value of the project, (6a), subject to the lender's ex ante zero profit condition, (6b), and the incentive constraint for effort, (6c).

We characterize the solution with the following lemmas.

LEMMA 1. *If $\sum_j q_j r_{\mathrm{f}}^j \geq k$, then the first best is attained. At a solution to problem* (6*a*):

(i) $P^1 = r_{\mathrm{f}}^1$, $P^2 = (1/q_2)\,[k - q_1 r_{\mathrm{f}}^1]$: *state-contingent debt is safe in all states.*

(ii) $\pi^{ij} = \pi_{\mathrm{FB}}^j$, $\forall^j$.

Proof. If $\sum_j q_j r_{\mathrm{f}}^j \geq k$, then P^1 and P^2, defined in (i), provide the lender with a zero expected profit and are paid with probability one in the relevant state: $P^j \leq r_{\mathrm{f}}^j$, $\forall_j$. Hence, the entrepreneur's effort, π^{ij}, in state Θ^j, solves $C'(\pi^{ij}) = r_{\mathrm{s}}^j - r_{\mathrm{f}}^j$, $\forall j$ (by (6c)), and the first-best level of effort $\pi^{ij} = \pi_{\mathrm{FB}}^j$ (by comparison with (5a)) is chosen for every Θ^j. Q.E.D.

Note that because $r_{\mathrm{f}}^1 < k$, we have $P^2 > P^1$ (by(i)).

LEMMA 2. *If $\sum_j q_j r_{\mathrm{f}}^j < k$, then the first best cannot be attained. P^j and π^{ij}, j = 1, 2, that solve problem* (6a) *satisfy*:

(i) $P^2 > P^1$, $r_{\mathrm{f}}^j < P^j < r_{\mathrm{s}}^j$, $\forall j$: *state-contingent debt is risky in all states.*

(ii) $\pi^{ij} < \pi_{\mathrm{FB}}^j$, $\forall j$: *the second best is attained.*

Proof. See Appendix A.

If $\sum_j q_j r_f^j < k$, then safe debt is infeasible in all states and hence the first-best effort level cannot be attained. The effort distortion is minimized by making the entrepreneur pay less in the bad state Θ^1; i.e., $P^1 < P^2$. The driving force is that the firm's revenues in the bad state Θ^1 are lower than those in the good state Θ^2, and therefore the entrepreneur's marginal return to effort in state Θ^1 is lower than that in state Θ^2 for any given face value of the debt. At the optimum the debt is risky in all states, i.e., $P^j > r_f^j$, $\forall j$, because effort costs are convex. This can be seen by considering a point (P^1, P^2) where the debt is safe in one of the two states, let us say Θ^2, and is risky in the other state, Θ^1, because of the lender's ex ante zero profit condition, (6b), and because $\sum_j q_j r_f^j < k$. Increasing P^2, so as to make the debt risky in Θ^2, and lowering P^1, so as to satisfy the lender's ex ante zero profit condition, is Pareto improving. Because the marginal cost of effort is increasing in effort, the efficiency gain that is attained in Θ^1, the state where at the original (P^1, P^2) point the debt was too risky and effort excessively low, outweighs the efficiency loss in Θ^2, the state where the first-best effort level was attained.

Lemmas 1 and 2 lead to:

PROPOSITION 1. *If states of nature can be contracted upon ex ante, then an optimal contract is a debt contract that specifies that the face value of the debt in state Θ^j is P^j, where P^j is defined in Lemmas* 1 *and* 2. *Hence, better states are allocated higher debt payments.*

This proposition asserts that allocating higher debt payments to better states makes debt safer and thereby reduces moral hazard. Moreover, contracting prior to the state realization dominates spot contracting. The reason is as follows. Since the lender is risk neutral, he is willing to accept ex ante a contract with nonnegative expected profit which imposes on him a loss in some states of nature (by Lemmas 1 and 2). Were an agreement to be reached after the state realization (spot lending), the lender would require to break even in *that* state and this would impose additional constraints on the entrepreneur's optimization problem.[4] Hence, regardless of the date that the funds are needed, the entrepreneur maximizes his payoff by signing the contract designed in Proposition 1 before the state is revealed. Of course, this is feasible only if state verification is possible. The next section analyzes the optimal contract when state verification is not possible.

[4] This is partly the intuition behind the result that spot lending is dominated by loan commitments in which terms are established before the state is revealed. See Boot *et al.* (1987) and Berkovitch and Greenbaum (1991).

IV. NONVERIFIABLE STATES

This section derives the main result of the paper: a debt contract with a warrant for the lender and a cash/equity settlement option for the borrower constitutes the ex ante optimal contract with unverifiable states. This contract reduces moral hazard because the better states are assigned the higher debt payments stated in Proposition 1, despite state unverifiability.

Assuming that state-contingent contracts are not possible, the results in Section III imply that we can focus on contracts of the form

$$\tau = \{k, (P^1); M\},$$

where k is the lender's transfer at $t = 0$, $(P^1) \equiv \min(r, P^1)$ is the initial debt, and M is a mechanism that allocates option rights to the parties to the contract. The parties' payoffs and the entrepreneur's choice of effort will depend on M. Solving the entrepreneur's optimization problem amounts to designing M so that the parties' payoffs are determined by the low debt, (P^1), in state Θ^1, and by the high debt, $(P^2) \equiv \min(r, P^2)$, in state Θ^2, where P^1, P^2 are defined in Proposition 1.[5]

A. *Mechanism Design*

We prove below that a (two-stage) mechanism that gives a warrant to the lender and a cash/equity settlement option to the borrower implements the appropriate allocation of high debt, (P^2), and low debt, (P^1). The warrant is designed so that it is *out* of the money in state Θ^1 and *in* the money in state Θ^2. If the state realization is Θ^2, the lender exercises the warrant and thereby forces the entrepreneur to choose between diluting his equity holdings and making the cash settlement. The entrepreneur chooses the latter and thereby makes the high debt payment, (P^2), that is required in state Θ^2. We now prove this formally. We begin with the following definitions:

(S_1, α) is the claim under which the entrepreneur:

(i) receives a transfer from the lender of $S_1 \geq 0$ at the interim date $0 < t < 1$ and

[5] Given the simplifying assumption that the probability distribution does not have the same support in both states, it is possible to design "forcing" contracts which implement the desired outcome. In a forcing contract, either party announces the state at the interim date and gets penalized if the return realization at $t = 2$ has a zero probability in the state that has been announced. Sufficient noise in the system can preclude a forcing contract with bounded penalities. Suppose r_s^j, r_f^j, $\forall j$, occur with exogenous probability in both states; then the penalities apply in the "wrong" state. Our solution is robust to noise. It is also robust to reinterpreting r_s^j as a reduced form of a stochastic model which has the same support in both states.

(ii) pays the lender

$$\min(r, P^1) + \alpha \max(r - P^1, 0)$$

at the terminal date $t = 2$, $0 \le \alpha \le 1$.

$E(W_E|(S_1, \alpha), \Theta^j)$ and $E(W_L|(S_1, \alpha), \Theta^j)$ are the conditional expected payoffs to the entrepreneur and lender, respectively, under (S_1, α) in state Θ^j. Recalling that $P^1 < r_s^1$ (by Proposition 1) and $r_s^1 < r_s^2$,

$$E(W_E|(S_1, \alpha), \Theta^j) = S_1 + (1 - \alpha)\,[(1 - \pi^{\alpha j}) \max(r_f^j - P^1, 0) + \pi^{\alpha j}(r_s^j - P^1)] - C(\pi^{\alpha j}), \tag{10}$$

$$E(W_L|(S_1, \alpha), \Theta^j) = -S_1 + (1 - \pi^{\alpha j}) \min(r_f^j, P^1) + \pi^{\alpha j} P^1 + \alpha[(1 - \pi^{\alpha j}) \max(r_f^j - P^1, 0) + \pi^{\alpha j}(r_s^j - P^1)], \tag{11}$$

where $\pi^{\alpha j}$ is the entrepreneur's effort in (S_1, α) in state Θ^j, $j = 1, 2$,

$$\pi^{\alpha j}\text{: } \arg\max_{\pi\alpha j} \{E(W_E|(S_1, \alpha), \Theta^j)\}. \tag{12}$$

(S_1, α) is the claim that determines the parties' payoffs when a *warrant* attached to the initial debt, (P^1), with an exercise date $0 < t < 1$, has been exercised and the entrepreneur has given the lender the fraction α of total equity in return for the exercise price S_1.[6] Note that as α increases, the entrepreneur's effort, $\pi^{\alpha j}$, decreases and thereby the project value shrinks. We can now define an optimal (two-stage) mechanism, M^*, as

(i) At some date before $t = 1$, the lender chooses between the initial debt (P^1) and (S_1, α).

(ii) If (S_1, α) has been selected, then the entrepreneur can either accept or opt for (P^2).

M^* attaches a warrant for the lender and a cash/equity settlement option for the borrower to the initial debt, (P^1). A choice of (S_1, α) is equivalent to the lender exercising the warrant. When the warrant is exercised, the entrepreneur either: (a) gives the lender the fraction α of total equity in return for S_1 or (b) makes the cash settlement $P^2 - P^1$ at date $t = 2$. If the entrepreneur chooses (a), he dilutes his equity holdings, and the existing claim becomes (S_1, α). An entrepreneur opting for (P^2) chooses the cash settlement, and the total amount he effectively pays at date $t = 2$ is $\min(r, P^2)$.

M^* is illustrated in Fig. 1, where $E(W_L|P^i, \Theta^j)$ and $E(W_E|P^i, \Theta^j)$, $i = 1$,

[6] With no dividend covenants, the exercise price, $S_1 \ge 0$, is distributed as dividends. The firm's cash flow net of the debt obligation (P^1) at date $t = 2$ is then $\max(r - P^1, 0)$.

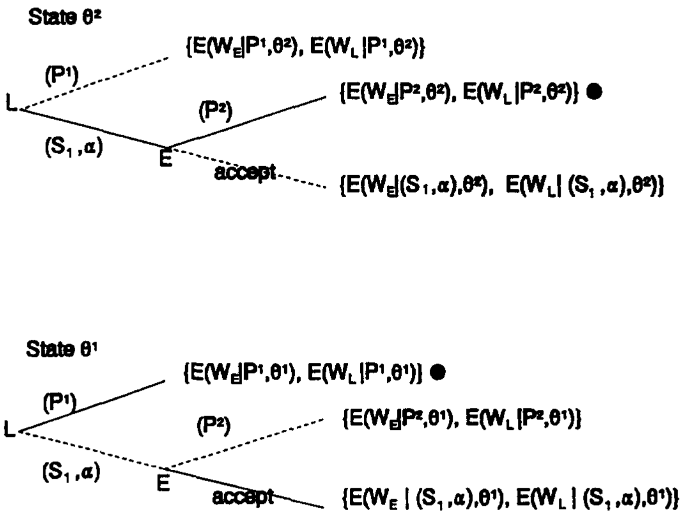

FIG. 1. Mechanism M^*. E, entrepreneur; L, lender.

$2, j = 1, 2$, are the conditional expected payoffs to the lender and entrepreneur, respectively, under debt (P^i) in state Θ^j. These are given by (8) and (9), respectively, once P^j is replaced by P^i, $i = 1, 2$.

We wish to establish the optimality of M^*. We define below the lender's strategy, s_L, and the entrepreneur's strategy, s_E, that implement the appropriate allocation of high debt, (P^2), and low debt, (P^1):

$s_L = \{$in state Θ^2 choose (S_1, α), in state Θ^1 choose $(P^1)\}$,

$s_E = \{$if the lender chooses (S_1, α) then opt for (P^2) in state Θ^2, accept in state $\Theta^1\}$.

The next lemma states the condition required for $\{s_L, s_E\}$ to be an equilibrium to the game defined by M^*. Define

$$\Omega_{E,\Theta 1} \equiv \{(S_1, \alpha) : E(W_E|P^2, \Theta^1) \leq E(W_E|(S_1, \alpha), \Theta^1),\ 0 \leq \alpha \leq 1,\ S_1 \geq 0\}$$

$$\Omega_L \equiv \{(S_1, \alpha) : E(W_L|P^1, \Theta^1) \geq E(W_L|(S_1, \alpha), \Theta^1),\ 0 \leq \alpha \leq 1,\ S_1 \geq 0\}$$

$$\Omega_{E,\Theta 2} \equiv \{(S_1, \alpha) : E(W_E|P^2, \Theta^2) \geq E(W_E|(S_1, \alpha), \Theta^2),\ 0 \leq \alpha \leq 1,\ S_1 \geq 0\}.$$

$\Omega_{E,\Theta 1}$ is the set of (S_1, α), where the entrepreneur in state Θ^1 prefers (S_1, α) to the debt (P^2): the entrepreneur is better off by making the equity settlement in state Θ^1. Ω_L is the set where the lender in state Θ^1 prefers the initial debt, (P^1), to (S_1, α). Finally, $\Omega_{E,\Theta 2}$ is the set where the entrepreneur in state Θ^2 prefers the debt (P^2) to (S_1, α): the entrepreneur is better off by making the cash settlement in state Θ^2.

LEMMA 3 (subgame perfect implementation). *Let* $(S_1, \alpha) \in \Omega_{E,\Theta 1} \cap \Omega_L \cap \Omega_{E,\Theta 2}$; *then the strategy pair* $\{s_L, s_E\}$ *is an equilibrium assessment in* M^*.

Proof. By inspection of Fig. 1, the following inequalities ensure that the entrepreneur's and the lender's strategies in Θ^1 are best responses to each other:

$$E(W_E|P^2, \Theta^1) \leq E(W_E|(S_1, \alpha), \Theta^1) \tag{13}$$

$$E(W_L|P^1, \Theta^1) \geq E(W_L|(S_1, \alpha), \Theta^1) \tag{14}$$

$\Omega_{E,\Theta 1}$ satisfies (13) and Ω_L satisfies (14).

Considering Θ^2, opting for (P^2) is the best response for the entrepreneur given s_L provided

$$E(W_E|P^2, \Theta^2) \geq E(W_L|(S^1, \alpha), \Theta^2), \tag{15}$$

since $E(W_L|P^2, \Theta^2) > E(W_L|P^1, \Theta^2)$ (by Lemmas 1 and 2); if (15) is satisfied, then s_L is the lender's best choice. $\Omega_{E,\Theta 2}$ satisfies (15). Q.E.D.

Note that in Lemma 3, $\Omega_{E,\Theta 1} \cap \Omega_L$ is the set of warrants that are *out* of the money in state Θ^1. Let $(S_1, \alpha) \in \Omega_{E,\Theta 1} \cap \Omega_L$. Here, if the warrant is exercised in state Θ^1, the entrepreneur dilutes his equity holdings and makes the lender worse off with respect to the initial debt, (P^1). $\Omega_{E,\Theta 2}$ is the set of warrants that is *in* the money in state Θ^2. For $(S_1, \alpha) \in \Omega_{E,\Theta 2}$, the lender exercising the warrant in state Θ^2 gets the entrepreneur to pay (P^2). Lemma 3 has then established that giving the lender a warrant that is *out* of the money in state Θ^1 and *in* the money in state Θ^2 implements the optimal allocation of low debt, (P^1), and high debt, (P^2). It remains to be determined whether such a warrant exists. We do this next.

LEMMA 4.

$$\Omega_{E,\Theta 1} \cap \Omega_L \cap \Omega_{E,\Theta 2} \neq \phi.$$

Proof. See Appendix B.

Figure 2 illustrates the feasible set for (S_1, α), $\Omega_{E,\Theta 1} \cap \Omega_L \cap \Omega_{E,\Theta 2}$. In Fig. 2, A is the schedule along which the lender in state Θ^1 is indifferent between (S_1, α) and the initial debt (P^1); i.e., (14) is satisfied with equality. Any point (S_1, α), with $S_1 \geq 0$ and $0 \leq \alpha \leq 1$, on and above the A schedule belongs to Ω_L. Along the B schedule, the entrepreneur in state Θ^1 is indifferent between (S_1, α) and the debt (P^2); i.e., (13) is satisfied with equality. Any point (S_1, α), with $S_1 \geq 0$ and $0 \leq \alpha \leq 1$, on and above the B

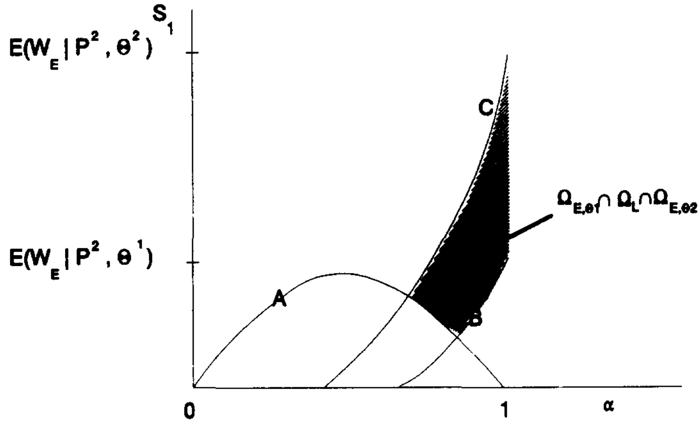

FIG. 2. The feasible set for (S_1, α).

schedule belongs to $\Omega_{E,\Theta 1}$. Along the C schedule, the entrepreneur is indifferent between (S_1, α) and the debt (P^2) in state Θ^2; i.e., (15) is satisfied with equality.

Any point (S_1, α), with $S_1 \geq 0$ and $0 \leq \alpha \leq 1$, on and below the C schedule belongs to $\Omega_{E,\Theta 2}$. Hence, the shaded area depicts the feasible set, $\Omega_{E,\Theta 1} \cap \Omega_L \cap \Omega_{E,\Theta 2}$.[7] This set is not empty. With equity, the firm value shrinks. Then, as seen in Fig. 2, a warrant that is just in the money in state Θ^1 (an (S_1, α) that is on the A schedule) has an exercise price, S_1, that is not monotonically increasing in the number of shares of stock it buys. There exists an α_0 such that for $\alpha > \alpha_0$, S_1 decreases as α increases. This is the basis for $\Omega_{E,\Theta 1} \cap \Omega_L \neq \phi$, and warrants that are out of the money in state Θ^1 do exist. The revenues of the firm in the good state, Θ^2, are higher than those in the bad state, Θ^1; i.e., $E(W_E|P^2, \Theta^2) > E(W_E|P^2, \Theta^1)$. Hence, as seen in Fig. 2, there exists an (S_1, α) such that the entrepreneur prefers (S_1, α) in state Θ^1 and prefers (P^2) in state Θ^2. Warrants that are *in* the money in state Θ^2 and *out* of the money in state Θ^1 do exist.

This leads to:

PROPOSITION 2. *A debt contract, $\tau = \{k, (P^1); M\}$, with a warrant for the lender and a cash/equity settlement option for the entrepreneur implements the solution attained in an optimal state-contingent contract and thus solves the entrepreneur's optimization problem.*

[7] Figure 2 is drawn for $\sum_j q_j r_f^j \geq k$. In this case $E(W_L \mid P^1, \Theta^1) = r_f^1$ (by Lemma 1), and the A schedule intersects the horizontal axis at $\alpha = 1$. If $\sum_j q_j r_f^j < k$, then $E(W_L \mid P^1, \Theta^1) > r_f^1$ (by Lemma 2), the A schedule intersects the horizontal axis at $0 < \alpha < 1$, and there is still a shaded area.

B. *Discussion*

Attaching to the debt a warrant for the lender and a cash/equity settlement option for the borrower makes it possible to contract optimally ex ante on *unverifiable* states: debt obligations are adjusted to state realizations so that the entrepreneur's actions match the optimum.

This contract (a) gives both parties options and (b) the game it defines is played in stages: the choice available to the entrepreneur is contingent on the exercise decision of the lender. This sequential structure effectively reduces the degrees of freedom available to the lender because he must take into account the entrepreneur's response, i.e., the cash/equity settlement choice.[8]

V. ARBITRARY NUMBER OF STATES

The results derived here are not limited to the two-state case. By definition of the problem, the revenues of the firm are higher in better states. Therefore, better states are allocated higher debt payments, and Proposition 1 holds regardless of the number of states. The mechanism that implements the optimal allocation with an arbitrarily large number of unverifiable states is identical to that in the two-state case.[9] Each warrant is designed so that it is in the money for a given state and out of the money for lower states. The warrant which is designed for a given state is then exercised in that state and induces the entrepreneur to make the debt payment that an optimal state-contingent contract assigns to that state. The optimal resource allocation is thereby attained, and therefore, regardless of the number of states, a debt contract with warrants for the lender and cash/equity settlement options for the borrower resolves the moral hazard problem with unverifiable states.

VI. CONCLUDING REMARKS

We have shown that a debt contract with warrants for the lender and cash/equity settlement options for the borrower reduces moral hazard with unverifiable states of nature. This package makes it possible to con-

[8] Callable-convertible debt also gives both parties options. In fact, the entrepreneur can redeem the debt at a stated price over a prespecified period, and the lender has the option to swap debt for equity according to a prespecified conversion rule. But, once the lender has exercised his option, i.e., swapped debt for equity, there is no further choice available to the entrepreneur. This is the main difference from the optimal contract above.

[9] A formal proof is available from the author upon request.

tract optimally ex ante on *unverifiable* states: debt obligations are adjusted to state realizations so that debt is made safer. The entrepreneur's incentive to expend effort improves as a result. Thus, this paper extends our understanding of the role that warrants can play in contracting by showing that they can control the underinvestment problem with unverifiable states. This complements Green's (1984) observation that warrants can control the hidden asset-substitution moral hazard problem.

In order to focus on the incentive role of financing, we have assumed risk neutrality. With risk aversion, an optimal contract would trade off the incentive motive against risk aversion. Pure debt financing would not in general be optimal, but there would be further scope for warrants as a way to improve risk sharing.

A basic ingredient of our model is contract incompleteness. Models of contract incompleteness were originally investigated by Grossman and Hart (1986). Aghion and Bolton (1988), Kahn and Huberman (1988), and Hart and Moore (1989) deal with incompleteness as related to corporate finance. The mechanisms designed in this literature, however, usually fail to implement the solution that would be attained in an optimal state-contingent contract. This is puzzling since the parties observe the state.[10]

Underlying incomplete contracts is the literature on implementation. Moore and Repullo (1988) have shown that with informed agents, almost anything can be implemented as subgame perfect equilibria. However, the mechanisms are far from simple and realistic. Varian (1990) shows that simple practical mechanisms can be used for some classes of problems. Varian's mechanisms require the presence of a third party who imposes a penalty if the two parties disagree on the state. We use the concept of subgame perfect equilibria as well, but the mechanisms that we have constructed are simple and realistic and do not require the presence of a third party to impose penalities.

Finally, we have designed mechanisms which implement efficient outcomes when both parties are informed. It would be useful to have mechanisms that work when the state of nature is privately revealed to the entrepreneur. We believe that warrants would still be part of the mechanism, but a complete answer awaits further research.

APPENDIX A: PROOF OF LEMMA 2

A. *Proof of (i) of Lemma 2*

CLAIM A1. *At a solution to problem* (6*a*), P^j *satisfies*

[10] Furthermore, little attempt has been made to design mechanisms that work when the first best cannot be attained. Without restricting the analysis to first-best allocations, we have designed mechanisms that work even when the first best cannot be attained.

$$P^j < r_s^j, \qquad \forall j, \tag{A1}$$

$$\min(r_f^j, P^j) = r_f^j, \qquad \forall_j. \tag{A2}$$

Proof of Claim A1. Equation (A1) follows because for $P^j \geq r_s^j$ the entrepreneur provides zero effort: $\pi^{jj} = 0$ (by the incentive constraint (6c)). To see (A2), suppose $P^1 < r_f^1$. Then, because $\sum_j q_j r_f^j < k$, the debt repayment that provides the lender a zero expected profit is $P^2 > r_f^2$ (debt is risky in Θ^2) and the entrepreneur's effort in Θ^2 falls below the first best. Increasing P^1 up to r_f^1, thereby lowering P^2 so to keep the lender's profit at zero, generates no distortion in Θ^1 and improves efficiency in Θ^2. The same reasoning applies for $P^2 < r_f^2$. Hence (A2) holds. Q.E.D.

It remains to be proved that at the optimum, $P^j > r_f^j$, $\forall j$, and $P^2 > P^1$. Setting up the Lagrangean function for (the first-order condition analog to) (6a), making use of (A1) and (A2), and deriving the first-order conditions for P^j, π^{jj}, $j = 1, 2$, lead to the following conditions for a solution:

$$(P^1 - r_f^1)[C''(\pi^{22})\pi^{22}] = (P^2 - r_f^2)[C''(\pi^{11})\pi^{11}], \tag{A3}$$

$$C'(\pi^{jj}) = r_s^j - P^j, \qquad \forall j, \tag{A4}$$

$$\sum_j q_j[\pi^{jj}P^j + (1 - \pi^{jj})r_f^j] = k. \tag{A5}$$

Equation (A3) is the condition that the locus of (P^1, P^2) points along which the project value is constant is tangent to the lender's zero profit curve. Equation (A4) is the first-order condition for the choice of effort π^{jj} in state Θ^j. Equation (A5) is the lender's zero profit condition; in (A5), $P^j \equiv \min(r_s^j, P^j)$ (by (A1)), and $r_f^j \equiv \min(r_f^j, P^j)$ (by (A2)).

Existence of a solution is by assumption: r_s^j are sufficiently high that the project has positive net present value, and $C(\pi^{jj})$ satisfies (3). Then there exist $P^j < r_s^j$, $\forall j$, and $0 < \pi^{jj} < 1$, $\forall j$, that satisfy the lender's zero profit condition (A5) and the incentive constraint (A4).

CLAIM A2.

$$P^j > r_f^j, \qquad \forall j.$$

Proof of Claim A2. Suppose $P^1 = r_f^1$; then $\pi^{11} = \tau_{FB}^1$ (by (A4)), $C''(\pi^{11})\pi^{11} > 0$ (by (3)), and $P^2 > r_f^2$ (by (A5) and $\sum_j q_j r_f^j < k$). The right-hand side of (A3) is positive and the left-hand side is zero. The same reasoning applies for $P^2 = r_f^2$. Hence, because $P^j \geq r_f^j$ (by (A2)), at the optimum, $P^j > r_f^j$, $\forall j$ (state-contingent debt is risky in all states). Q.E.D.

CLAIM A3.

$$P^2 > P^1.$$

Proof of Claim A3. Suppose on the contrary that $P^1 \geq P^2$. Then

$$P^2 - r_f^2 \leq P^1 - r_f^1 \tag{A6}$$

$$C''(\pi^{11})\pi^{11} < C''(\pi^{22})\pi^{22} \tag{A7}$$

and (A3) fails to be satisfied.

Because $r_f^1 \leq r_f^2$ (by (4b)), $P^2 \leq P^1$ implies (A6). $C''(\,)$ is nondecreasing in effort, and thus (A7) holds if $\pi^{11} < \pi^{22}$. Effort, π^{jj}, satisfies (A4) and $C(\,)$ is convex. Hence $\pi^{11} < \pi^{22}$ if

$$r_s^1 - P^1 < r_s^2 - P^2 \tag{A7$'$}$$

because $r_s^2 > r_s^1$ (by (4a)), $P^1 \geq P^2$ implies (A7′). Hence, $\pi^{11} < \pi^{22}$: (A7) holds and condition (A3) fails to be satisfied.

To see that $P^2 > P^1$ satisfies (A3), consider the situation where $P^2 = P^1$. Then, because of the argument above, $(P^1 - r_f^1)\,[C''(\pi^{22})\pi^{22}] > (P^2 - r_f^2)\,[C''(\pi^{11})\pi^{11}]$. Suppose we increase P^2 and lower P^1 so as to keep the lender at zero profit. Then $(P^2 - r_s^2)\,[C''(\pi^{11})\pi^{11}]$ increases and $(P^1 - r_f^1)\,[C''(\pi^{22})\pi^{22}]$ decreases. Therefore, by continuity, there exist P^2 and P^1, with $P^2 > P^1$, that satisfy (A3). Q.E.D.

(i) of Lemma 2 is proved.

B. *Proof of (ii) of Lemma 2*

π^{jj} solves the incentive constraint (A4), π^j_{FB} solves (5a), $C(\,)$ is convex (by (3)), and $P^j > r_f^j$, $\forall j$. Hence, $\pi^{jj} < \pi^j_{FB}$, $\forall j$, and there is underprovision of effort in all states.

Lemma 2 is then proved.

APPENDIX B: PROOF OF LEMMA 4

We have to prove that there exists an (S_1, α) that solves

$$E(W_E|P^2, \Theta^2) \geq E(W_E|(S_1, \alpha), \Theta^2) \tag{B1}$$

$$E(W_E|P^2, \Theta^1) \leq E(W_E|(S_1, \alpha), \Theta^1) \tag{B2}$$

$$E(W_L|P^1, \Theta^1) \geq E(W_L|(S_1, \alpha), \Theta^1) \tag{B3}$$

$$0 \leq \alpha \leq 1 \tag{B4}$$

$$S_1 \geq 0. \tag{B5}$$

CLAIM B1.

$$E(W_E|P^2, \Theta^2) > E(W_E|(P_2, \Theta^1). \tag{B6}$$

Proof of Claim B1. $E(W_E|P^2, \Theta^2)$ is given by (9), and $E(W_E|P^2, \Theta^1) = (1 - \pi^{21}) \max(r_f^1 - P^2, 0) + \pi^{21} \max(r_s^1 - P^2, 0) - C(\pi^{21})$, where

$$\pi^{21}: \arg\max_{\pi^{21}} \{E(W_E|P^2, \Theta^1)\}.$$

$r_f^1 < P^2 < r_s^2$ (by Proposition 1 and inequalities (4b) and (4c)) and $r_s^2 > r_s^1$ (by (4a)); then (B6) holds. Q.E.D.

CLAIM B2.

$$E(W_L|P^1, \Theta^1) \geq r_f^1. \tag{B7}$$

Proof of Claim B2. Now, $r_f^1 \leq P^1 < r_s^1$ (by Proposition 1). Then, $E(W_L|P^1, \Theta^1) = (1 - \pi^{11})r_f^1 + \pi^{11}P^1$, where π^{11} solves (6c). Hence, $0 < \pi^{11} < 1$ (by (3)) and (B7) holds. Q.E.D.

Set α equal (close to) one. Because the entrepreneur provides zero effort, it follows from (10) and (11) that

$$E(W_E|(S_1, 1), \Theta^j) = S_1, \qquad \forall j, \tag{B8}$$

$$E(W_L|(S_1, 1), \Theta^1) = -S_1 + r_f^1. \tag{B9}$$

Substituting (B8) into (B2) and (B9) into (B3) leads to

$$E(W_E|P^2, \Theta^1) \leq S_1 \tag{B2$'$}$$

$$E(W_L|P^1, \Theta^1) \geq -S_1 + r_f^1. \tag{B3$'$}$$

Now, keep α equal (close to) one and set S_1 at (close to) the value that satisfies (B1) with equality:

$$S_1 = E(W_E|P^2, \Theta^2). \tag{B1$'$}$$

Then, because of (B6), inequality (B2′) is satisfied. $r_s^j > P^j$, $\forall j$; thus $E(W_E|P^2, \Theta^2) > 0$ and S_1 is strictly positive. Then, because $E(W_L|P^1, \Theta^1) \geq r_f^1$ (by (B7)), inequality (B3′) is satisfied. Thus, there exists an (S_1, α) that solves (B1)–(B5) and Lemma 4 is proved.

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